Weighting Methodology of Equity Performance Attribution

Morningstar Methodology Paper
December 5, 2008

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Introduction

The Morningstar® Equity Performance Attribution Methodology document describes the foundations of performance attribution and defines the primary methodology recommended by Morningstar. This document is a supplement to the main methodology, presenting how the holdings weights of the portfolio and the benchmark are calculated under various exception cases. In an ideal situation where an attribution analysis is most meaningful, portfolio and benchmark holdings are reported more frequently. There are many benefits in having more frequent holdings data including reducing the Return Gap, the portion of the return that cannot be explained and is usually caused by intra-period portfolio transactions. Another important benefit is providing the ability to select any range of dates for analysis while using actual holdings information. There are times portfolio or benchmark holdings may not be available on desired dates, and assumptions must be made in order to infer holdings weights based on obtainable holdings data from neighboring time periods. This document outlines the assumptions and methodology for inferring holdings weights. It also addresses circumstances where data are not perfect, such as handling of securities that are not recognized by the database or are missing returns in some periods. The methodology accounts for the rare possibility that holdings are incomplete, resulting in a portfolio or benchmark with holdings weights that add up to less than 100%.

First, this paper will expand on the Single Period definition described in the Morningstar® Equity Performance Attribution Methodology document to accommodate for user-specified time periods that may not begin on a date that has holdings data. When holdings information is not available on a desired date, it is inferred from a neighboring period. The next section presents an overview of inferring holdings weights and provides examples. It is followed by a section that is dedicated to the methodology of inferring holdings without excluding the exemption securities described above. The following section focuses on re-scaling holdings to handle exception securities. The next section presents a methodology that calculates average multiple single-period holdings weights. It is a common industry practice to display holdings weights in average numbers even though these average weights are not used in an actual attribution analysis. The last section addresses special situations such as performing attribution on a fund of fund or an aggregate portfolio.

Determining Single Periods

Overview

As defined in the Morningstar® Equity Performance Attribution Methodology document, a single holding period is the time between reported portfolio holdings. For example, if portfolio holdings are available on 10/31/2000, 4/30/2001, and 7/31/2001, the period from 11/1/2000 to 4/30/2001 represents the first holding period, and the period from 5/1/2001 to 7/31/2001 is the second holding period. This section will further expand this methodology as one might prefer to perform an attribution analysis with beginning (and ending) dates where portfolio holdings are not available. This methodology presents two methods in determining single periods, and the method depends on whether more frequent display of sub-period information is required.

As a general rule, single periods are determined by portfolio holdings dates. It does not matter whether benchmark holdings are available on these dates, as it is the portfolio that is in the center-stage of an attribution analysis. However, benchmark holdings ought to be available on the same dates in order to achieve the most meaningful analysis.

Single Display

This is the recommended method, and it is applicable when one desires to perform an attribution analysis over a specified overall period and demands no other display periods. The alternative method is used when regular sub-periods of this overall analysis period is required.

Under the single display methodology, single periods are determined by the beginning date and the ending date of attribution analysis and dates where portfolio holdings are available. This methodology is very similar to the single period methodology stated in the Morningstar® Equity Performance Attribution Methodology document, with the only difference being the need to accommodate for beginning (and ending) dates that do not fall on days where portfolio holdings are available. For example, selecting the time period from 9/1/2000 to 10/31/2001, the following would be the breakdown of single periods and required holdings data:

#	Single Periods	Weights Based On	Holdings Data
1	09/01/2000 to 10/31/2000	08/31/2000	Inferred
2	11/01/2000 to 04/30/2001	10/31/2000	Actual
3	05/01/2001 to 07/31/2001	04/30/2001	Actual
4	08/01/2001 to 10/31/2001	07/31/2001	Actual

Since the chosen beginning date of the analysis is 9/1/2000, the portfolio holdings for 8/31/2000 are required for attribution analysis. When data is unavailable, holdings data from a nearby date are used in inferring what the portfolio holdings would be. The selection of a nearby date is described later in this document. Note that in the table above, actual holdings are used in all cases except the first time period where actual holdings are not available.

Multiple Sub-Period Display

This method is the alternative to the Single Display method described in the previous section, and is applicable to the scenario where one desires to perform an overall attribution analysis accompanied by sub-period details. In this situation, more single periods are required in order to match the cutoff dates of the sub-periods. For example, for the overall analysis period from 9/1/2000 to 10/31/2001, one might desire to examine sub-period attribution results by calendar quarter in addition to that of the whole period. In such a case, single periods are determined by calendar quarter-end in addition to the beginning date, the ending date of the analysis, and the dates when portfolio holdings are available. The following table illustrates the example:

#	Single Periods	Weights Based On	Holdings Data	Calendar Quarters
1	09/01/2000 to 09/30/2000	08/31/2000	Inferred	09/01/2000 to 09/30/2000
2	10/01/2000 to 10/31/2000	09/30/2000	Inferred	10/01/2000 to 12/31/2000
3	11/01/2000 to 12/31/2000	10/31/2000	Actual	10/01/2000 to 12/31/2000
4	01/01/2001 to 03/31/2001	12/31/2000	Inferred	01/01/2001 to 03/31/2001
5	04/01/2001 to 04/30/2001	03/31/2001	Inferred	04/01/2001 to 06/30/2001
6	05/01/2001 to 06/30/2001	04/30/2001	Actual	04/01/2001 to 06/30/2001
7	07/01/2001 to 07/31/2001	06/30/2001	Inferred	07/01/2001 to 09/30/2001
8	08/01/2001 to 09/30/2001	07/31/2001	Actual	07/01/2001 to 09/30/2001
9	10/01/2001 to 10/31/2001	09/30/2001	Inferred	10/01/2001 to 10/31/2001

Under this methodology the overall multi-period attribution result from 9/1/2000 to 10/31/2001 is the accumulation of nine single-period attribution results. In addition to the overall period, attribution results of six calendar quarters will be presented, and they are listed in the last column of the table above. Some calendar quarters only have one single period each, such as the partial calendar quarter from 9/1/2000 to 9/30/2000. Some calendar quarters have multiple single periods, such as the calendar quarter from 10/1/2000 to 12/31/2000 which has two single periods (#2 and #3). In this case the calendar quarter is considered a multi-period, and its attribution result is the multi-period accumulation of those two single periods' attribution results. Instead of considering a calendar quarter like 10/1/2000 to 12/31/2000 as one single period, it is divided into two single-periods because portfolio holdings are available on 10/31/2000, and actual portfolio holdings data should to be used whenever they are available. In this example, the overall multi-period result is the accumulation of nine single periods even though six calendar quarters are displayed. The cumulative result of the nine single periods is the same as that of the six calendar quarters under the geometric multi-period method. These numbers could differ under the arithmetic multi-period method.

Inferring Holdings Weights

Overview

In a perfect world, a meaningful attribution analysis is performed when portfolio and benchmark holdings are always available. In reality, users of attribution analysis may decide to report on time periods with beginning dates that fall on days where holdings information is not available. When this happens, holdings data from a nearby date are used in inferring what the holdings would be on the desired date. The preferred method is to use holdings information from the desired date's most recent past to estimate holdings of the desired date, a forward inferring situation. In contrast, the alternative method is to use holdings information from the desired date's most recent future holdings to estimate that of the desired date, a backwards inferring situation. The preferred method is favored because it is consistent with the direction that time travels from past to present to future, as portfolio holdings from a future date is theoretically unknown as of the desired date.

Holdings weights are inferred based on the buy-and-hold assumption, in other words, no purchases or sales are made between the date where holdings information is available and the desired date that holdings information must be inferred. Thus, the list of holdings is the same between these two dates, and the only difference is the weights of individual securities which have grown or diminished by the relative return between this security and that of the rest of the holdings. In other words, in a forward inferring situation, if a security performed better than the rest of the portfolio between the holdings date and the desired date, its weight will increase. Similarly, if its return is worse than the rest of the portfolio between these two dates, its weight will decrease. If it has the same performance as the rest of the portfolio between these two dates, its weight will stay the same. In a backward inferring situation, the opposite happens because the holdings date is later than the desired date; thus, a better performing security's weight is diminished while inferring to the desired date.

When one chooses the forward inferring methodology, special attention needs to be given to the first few periods because holdings information may not be available prior to these periods to be used in inferring weights. In this situation, holdings from a later date must be used in inferring weights. In other words, weights are filled backward in time using the backward inferring methodology even though the general direction of the methodology is to infer forward in time. Similarly, when one chooses the backward inferring methodology, special attention needs to be paid to the last few periods because holdings information may not be available later than these periods to be used in inferring weights. When this happens, holdings weights are filled forward in time using the forward inferring methodology even though the general direction of the methodology is to infer backward in time.

Inferring Holdings Weights (continued)

Illustration of Inferring Weights Forward In Time

Below is an example of the concepts described above. For this example, assume that benchmark holdings data are available on 11/30/2000, 2/28/2001, 5/31/2001, and 8/31/2001. As demonstrated in the Determining Single Periods section of this document, nine single periods are created based on portfolio holdings dates. These nine single periods are illustrated in the tables below, followed by explanations.

Inferring Portfolio Holdings Forward In Time:

#	Single Periods	Weights Based On	Infer From	Returns Used In Inferring
1	09/01/2000 to 09/30/2000	08/31/2000	10/31/2000*	09/01/2000 to 10/31/2000
2	10/01/2000 to 10/31/2000	09/30/2000	10/31/2000*	10/01/2000 to 10/31/2000
3	11/01/2000 to 12/31/2000	10/31/2000	Actual	
4	01/01/2001 to 03/31/2001	12/31/2000	10/31/2000	11/01/2000 to 12/31/2000
5	04/01/2001 to 04/30/2001	03/31/2001	10/31/2000	11/01/2000 to 03/31/2001
6	05/01/2001 to 06/30/2001	04/30/2001	Actual	
7	07/01/2001 to 07/31/2001	06/30/2001	04/30/2001	05/01/2001 to 06/30/2001
8	08/01/2001 to 09/30/2001	07/31/2001	Actual	
9	10/01/2001 to 10/31/2001	09/30/2001	07/31/2001	08/01/2001 to 09/30/2001

Inferring Benchmark Holdings Forward In Time:

#	Single Periods	Weights Based On	Infer From	Returns Used In Inferring
1	09/01/2000 to 09/30/2000	08/31/2000	11/30/2000*	09/01/2000 to 11/30/2000
2	10/01/2000 to 10/31/2000	09/30/2000	11/30/2000*	10/01/2000 to 11/30/2000
3	11/01/2000 to 12/31/2000	10/31/2000	11/30/2000*	11/01/2000 to 11/30/2000
4	01/01/2001 to 03/31/2001	12/31/2000	11/30/2000	12/01/2000 to 12/31/2000
5	04/01/2001 to 04/30/2001	03/31/2001	02/28/2001	03/01/2001 to 03/31/2001
6	05/01/2001 to 06/30/2001	04/30/2001	02/28/2001	03/01/2001 to 04/30/2001
7	07/01/2001 to 07/31/2001	06/30/2001	05/31/2001	06/01/2001 to 06/30/2001
8	08/01/2001 to 09/30/2001	07/31/2001	05/31/2001	06/01/2001 to 07/31/2001
9	10/01/2001 to 10/31/2001	09/30/2001	08/31/2001	09/01/2001 to 09/30/2001

Inferring Holdings Weights (continued)

The first two columns of the tables displays the nine single periods, and the Weights Based On column demonstrates the dates holdings data are required for attribution analysis. These dates are selected because attribution analyses use beginning of period weights. The Infer From columns shows whether actual holdings weights are available. If actual data are not available, the column indicates the date holdings should be inferred from. Note that dates in the Infer From column are usually earlier than those in the Weights Based On column because the example illustrates inferring weights forward in time. The exceptions are the first few single periods, marked with asterisks, which must be filled backward due to the lack of past holdings data. The last column, the Returns Used In Inferring columns shows the inferring weight time periods, and returns from this period are used in inferring weights. As discussed above, each security's inferred weight depends on the security's performance relative to those of others during this period. The inferring weight formulas are presented in later sections.

Inferring Holdings Weights (continued)

Illustration of Inferring Holdings Weights Backward In Time

Using the same example, the following tables illustrate the backward inferring methodology. The interpretation of these tables is the same as the equivalent tables for inferring weights forward in time. Note that the dates in the Infer From column are usually later than those in the Weights Based On column because the example illustrates inferring weights backward in time. The exception is the last single period, marked with an asterisk, which must be filled forward due to the lack of future holdings data.

Inferring Portfolio Holdings Backward In Time:

#	Single Periods	Weights Based On	Infer From	Returns Used In Inferring
1	09/01/2000 to 09/30/2000	08/31/2000	10/31/2000	09/01/2000 to 10/31/2000
2	10/01/2000 to 10/31/2000	09/30/2000	10/31/2000	10/01/2000 to 10/31/2000
3	11/01/2000 to 12/31/2000	10/31/2000	Actual	
4	01/01/2001 to 03/31/2001	12/31/2000	04/30/2001	01/01/2001 to 04/30/2001
5	04/01/2001 to 04/30/2001	03/31/2001	04/30/2001	04/01/2001 to 04/30/2001
6	05/01/2001 to 06/30/2001	04/30/2001	Actual	
7	07/01/2001 to 07/31/2001	06/30/2001	07/31/2001	07/01/2001 to 07/31/2001
8	08/01/2001 to 09/30/2001	07/31/2001	Actual	
9	10/01/2001 to 10/31/2001	09/30/2001	07/31/2001*	08/01/2001 to 09/30/2001

Inferring Benchmark Holdings Backward In Time:

#	Single Periods	Weights Based On	Infer From	Returns Used In Inferring
1	09/01/2000 to 09/30/2000	08/31/2000	11/30/2000	09/01/2000 to 11/30/2000
2	10/01/2000 to 10/31/2000	09/30/2000	11/30/2000	10/01/2000 to 11/30/2000
3	11/01/2000 to 12/31/2000	10/31/2000	11/30/2000	11/01/2000 to 11/30/2000
4	01/01/2001 to 03/31/2001	12/31/2000	02/28/2001	01/01/2001 to 02/28/2001
5	04/01/2001 to 04/30/2001	03/31/2001	05/31/2001	04/01/2001 to 05/31/2001
6	05/01/2001 to 06/30/2001	04/30/2001	05/31/2001	05/01/2001 to 05/31/2001
7	07/01/2001 to 07/31/2001	06/30/2001	08/31/2001	07/01/2001 to 08/31/2001
8	08/01/2001 to 09/30/2001	07/31/2001	08/31/2001	08/01/2001 to 08/31/2001
9	10/01/2001 to 10/31/2001	09/30/2001	08/31/2001*	09/01/2001 to 09/30/2001

Inferring Weights without Excluding Exception Securities

Overview

There are circumstances where the data is not perfect, such as securities that are not recognized by the database or are missing returns in some periods. It is rare but possible that weights might not add up to 100%. While some of these securities with exceptions ought to be eliminated from an attribution analysis for their lack of meaningful information, they do not need to be excluded if one desires to look at holdings weights. Some reports show these weights next to the weights used for attribution analysis, and some reports show these weights alone.

Inferring Portfolio Weights Forward In Time

If actual weights are available, they are displayed as-is. If actual weights are unavailable, they are inferred forward in time with the formula below. The formula consists of two conditions. The first condition is applied when the security's return is missing during the inferring weight period. In this case, this security's weight is carried forward as-is, implicitly assuming that this security performed the same as the rest of the portfolio during the inferring weight period. The second condition in the formula infers a security's weight forward in time by comparing its return to that of the rest of the portfolio while adjusting for the fact that the weights of the securities with missing returns must be held constant.

[1]

$$fiw^P_{s,t} = \begin{cases} W^P_{s,d} & \text{if s has incomplete perf in the infer wt period} \\ \dfrac{W^P_{s,d}(1+R^P_{s,d,t})}{\sum_{j=1}^{c} W^P_{j,d}(1+R^P_{j,d,t})}\left(\sum_{i=1}^{n} W^P_{i,d} - \sum_{k=1}^{m} W^P_{k,d}\right) & \text{if s has complete perf in the infer wt period} \end{cases}$$

Where:

$fiw^P_{s,t}$	=	forward-inferred portfolio weight for stock s at period t, where t is from the Weights Based On column
$W^P_{s,d}$	=	original portfolio weight of stock s at period d, where d is the date in the Infer From column
$R^P_{s,d,t}$	=	portfolio return for stock s in the inferring weight period from d to t, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the portfolio with complete performance in the inferring weight period (d to t)
n	=	number of securities in the portfolio, where $n = c + m$
m	=	number of securities in the portfolio with incomplete performance in the inferring weight period (d to t)

Note:

- This formula does not apply to the first two single periods where weights need to be filled backward using formula [3].

Inferring Benchmark Weights Forward In Time

The methodology for inferring benchmark holdings weights is similar to that of inferring portfolio holdings weights. If actual weights are available, they are displayed as-is. If actual weights are unavailable, benchmark holdings weights are inferred forward in time with the formula below:

[2]

$$fiw_{s,t}^{B} = \begin{cases} W_{s,d}^{B} & \text{if s has incomplete perf in the infer wt period} \\ \dfrac{W_{s,d}^{B}(1+R_{s,d,t}^{B})}{\sum_{j=1}^{c} W_{j,d}^{B}(1+R_{j,d,t}^{B})}\left(\sum_{i=1}^{n} W_{i,d}^{B} - \sum_{k=1}^{m} W_{k,d}^{B}\right) & \text{if s has complete perf in the infer wt period} \end{cases}$$

Where:

$fiw_{s,t}^{B}$	=	forward-inferred benchmark weight for stock s at period t, where t is from the Weights Based On column
$W_{s,d}^{B}$	=	original benchmark weight of stock s at period d, where d is the date in the Infer From column
$R_{s,d,t}^{B}$	=	benchmark return for stock s in the inferring weight period from d to t, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the benchmark with complete performance in the inferring weight period (d to t)
n	=	number of securities in the benchmark, where $n = c + m$
m	=	number of securities in the benchmark with incomplete performance in the inferring weight period (d to t)

Note:

- This formula does not apply to the first three single periods where weights need to be filled backward using formula [4].

Inferring Portfolio Weights Backward In Time

The methodology for inferring portfolio holdings weights backward in time is similar to that of inferring portfolio holdings weights forward in time, but the exponent in the returns portion of the formula is negative as it is the inverse of inferring forward in time. If actual weights are available, they are displayed as-is. If actual weights are unavailable, portfolio holdings weights are inferred backward in time with the formula below:

[3]

$$biw_{s,t}^{P} = \begin{cases} W_{s,d}^{P} & \text{if s has incomplete perf in the infer wt period} \\ \dfrac{W_{s,d}^{P}(1+R_{s,d,t}^{P})^{-1}}{\sum_{j=1}^{c} W_{j,d}^{P}(1+R_{j,d,t}^{P})^{-1}} \left(\sum_{i=1}^{n} W_{i,d}^{P} - \sum_{k=1}^{m} W_{k,d}^{P}\right) & \text{if s has complete perf in the infer wt period} \end{cases}$$

Where:

$biw_{s,t}^{P}$	=	backward-inferred portfolio weight for stock s at period t, where t is from the Weights Based On column
$W_{s,d}^{P}$	=	original portfolio weight of stock s at period d, where d is the date in the Infer From column
$R_{s,t,d}^{P}$	=	portfolio return for stock s in the inferring weight period from t to d, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the portfolio with complete performance in the inferring weight period (t to d)
n	=	number of securities in the portfolio, where $n = c + m$
m	=	number of securities in the portfolio with incomplete performance in the inferring weight period (t to d)

Note:

- This formula does not apply to the last single period where weights need to be filled forward using formula [1].

Inferring Benchmark Weights Backward In Time

The methodology for inferring benchmark holdings weights is similar to that of inferring portfolio holdings weights. If actual weights are available, they are displayed as-is. If actual weights are unavailable, benchmark holdings weights are inferred backward in time with the formula below:

[4]

$$biw_{s,t}^{B} = \begin{cases} W_{s,d}^{B} & \text{if s has incomplete perf in the infer wt period} \\ \dfrac{W_{s,d}^{B}(1+R_{s,d,t}^{B})^{-1}}{\sum_{j=1}^{c} W_{j,d}^{B}(1+R_{j,d,t}^{B})^{-1}} \left(\sum_{i=1}^{n} W_{i,d}^{B} - \sum_{k=1}^{m} W_{k,d}^{B}\right) & \text{if s has complete perf in the infer wt period} \end{cases}$$

Where:

$biw_{s,t}^{B}$	=	backward-inferred benchmark weight for stock s at period t, where t is from the Weights Based On
$W_{s,d}^{B}$	=	original benchmark weight of stock s at period d, where d is the date in the Infer From column
$R_{s,t,d}^{B}$	=	benchmark return for stock s in the inferring weight period from t to d, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the benchmark with complete performance in the inferring weight period (t to d)
n	=	number of securities in the benchmark, where $n = c + m$
m	=	number of securities in the benchmark with incomplete performance in the inferring weight period (t to d)

Note:

- This formula does not apply to the last single period where weights need to be filled forward using the formula [2].

Inferring Weights and Re-Scaling to Exclude Exception Securities

Overview

There are circumstances where the data is not perfect, such as securities that are not recognized by the database or are missing returns in the attribution analysis periods. There is also the rare possibility that holdings are incomplete, resulting in a portfolio or benchmark with holdings weights that add up to less than 100%. These securities with exceptions should be eliminated from an attribution analysis for their lack of meaningful information, and non-equity securities are also excluded because this methodology pertains to equity securities only. When these exception securities are excluded from the portfolio or benchmark, the weights of remaining securities must be re-scaled to 100%. The methodology for re-scaling actual holdings weights is presented first, and it is followed by formulas for inferring weights while re-scaling.

Re-Scaling Actual Portfolio and Benchmark Weights

[5]

$$rw_{s,t}^{P} = \begin{cases} ignored & \text{if } s \text{ has incomplete perf in the attrib period} \\ & \text{or is a non-equity or unrecognized security} \\ \dfrac{W_{s,t}^{P}}{\sum_{i=1}^{n} W_{i,t}^{P}} & \text{if } s \text{ is a recognized equity security with complete perf in attrib period} \end{cases}$$

[6]

$$rw_{s,t}^{B} = \begin{cases} ignored & \text{if } s \text{ has incomplete perf in the attrib period} \\ & \text{or is a non-equity or unrecognized security} \\ \dfrac{W_{s,t}^{B}}{\sum_{j=1}^{m} W_{j,t}^{B}} & \text{if } s \text{ is a recognized equity security with complete perf in attrib period} \end{cases}$$

Where:

$rw_{s,t}^{P}$	=	re-scaled portfolio weight for stock s at period t
$rw_{s,t}^{B}$	=	re-scaled portfolio weight for stock s at period t
$W_{s,t}^{P}$	=	original weight of stock s in the portfolio at period t
n	=	number of recognized equity securities in the portfolio with complete performance in the attribution period
$W_{s,t}^{B}$	=	original weight of stock s in the benchmark at period t
m	=	number of recognized equity securities in the benchmark with complete performance in the attribution period

Inferring Weights and Re-Scaling to Exclude Exception Securities (continued)

Re-Scaling and Inferring Portfolio Weights Forward In Time

The formula consists of three conditions. The first condition states the types of securities that ought to be excluded from an attribution analysis for their lack of meaningful information. The remaining two conditions are for securities that fail the first condition. The second condition is applied when the security's return is missing during the inferring weight period. In this case, it is assumed that this security performed the same as the rest of the attribution portfolio, and the security's weight is simply re-scaled. The last condition in the formula re-scales a security's weight while inferring forward in time.

[7]

$$\begin{cases} firw^{P}_{s,t} = \text{ignored} & \text{if s has incomplete perf in attribution period or is a non - equity or unrecognized security} \\ \text{otherwise,} \quad firw^{P}_{s,t} = \dfrac{W^{P}_{s,d}}{\sum_{i=1}^{n} W^{P}_{i,d}} & \text{if s has incomplete perf in infer wt period} \\ \text{otherwise,} \quad firw^{P}_{s,t} = \dfrac{W^{P}_{s,d}(1+R^{P}_{s,d,t})}{\sum_{j=1}^{c} W^{P}_{j,d}(1+R^{P}_{j,d,t})}\left(1-\dfrac{\sum_{k=1}^{m} W^{P}_{k,d}}{\sum_{i=1}^{n} W^{P}_{i,d}}\right) & \text{if s has complete perf in infer wt period} \end{cases}$$

Where:

$firw^{P}_{s,t}$	=	forward-inferred re-scaled portfolio weight for stock s at period t, where t is the date in the Weights Based On column
$W^{P}_{s,d}$	=	original weight of stock s in the portfolio at period d, where d is the date in the Infer From column
$R^{P}_{s,d,t}$	=	portfolio return for stock s in the inferring weight period from d to t, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the portfolio excluding ignored securities and those with incomplete performance in the inferring weight period from d to t
n	=	number of securities in the portfolio except for ignored securities, where $n = c + m$
m	=	number of securities in the portfolio with incomplete performance in the inferring weight period from d to t but excluding ignored securities

Note:

- This formula does not apply to the first two single periods where weights need to be filled backward using formula [9].

Re-Scaling and Inferring Benchmark Weights Forward In Time

The methodology for inferring while re-scaling benchmark holdings weights is similar to that of inferring while re-scaling portfolio holdings weights. The formula is below:

[8]

$$\begin{cases} firw^{B}_{s,t} = \text{ignored} & \text{if s has incomplete perf in attribution period or is a non-equity or unrecognized security} \\ \text{otherwise,} \quad firw^{B}_{s,t} = \dfrac{W^{B}_{s,d}}{\sum_{i=1}^{n} W^{B}_{i,d}} & \text{if s has incomplete perf in infer wt period} \\ \text{otherwise,} \quad firw^{B}_{s,t} = \dfrac{W^{B}_{s,d}(1+R^{B}_{s,d,t})}{\sum_{j=1}^{c} W^{B}_{j,d}(1+R^{B}_{j,d,t})}\left(1-\dfrac{\sum_{k=1}^{m} W^{B}_{k,d}}{\sum_{i=1}^{n} W^{B}_{i,d}}\right) & \text{if s has complete perf in infer wt period} \end{cases}$$

Where:

$firw^{B}_{s,t}$	=	forward-inferred re-scaled benchmark weight for stock s at period t, where t is the date in the Weights Based On column
$W^{B}_{s,d}$	=	original weight of stock s in the benchmark at period d, where d is the date in the Infer From column
$R^{B}_{s,d,t}$	=	benchmark return for stock s in the inferring weight period from d to t, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the benchmark excluding ignored securities and those with incomplete performance in the inferring weight period from d to t
n	=	number of securities in the benchmark except for ignored securities, where $n = c + m$
m	=	number of securities in the benchmark with incomplete performance in the inferring weight period from d to t but excluding ignored securities

Note:

- This formula does not apply to the first three single periods where weights need to be filled backward using formula [10].

Re-Scaling and Inferring Portfolio Weights Backward In Time

The methodology for inferring portfolio holdings weights backward in time while re-scaling is similar to that of inferring portfolio holdings weights forward in time while re-scaling, but the exponent in the returns portion of the formula is negative as it is the inverse of inferring forward in time. The formula is below:

[9]

$$\begin{cases} birw_{s,t}^{P} = \text{ignored} & \text{if s has incomplete perf in attribution period or is a non - equity or unrecognized security} \\ \text{otherwise, } birw_{s,t}^{P} = \dfrac{W_{s,d}^{P}}{\sum_{i=1}^{n} W_{i,d}^{P}} & \text{if s has incomplete perf in infer wt period} \\ \text{otherwise, } birw_{s,t}^{P} = \dfrac{W_{s,d}^{P}(1+R_{s,t,d}^{P})^{-1}}{\sum_{j=1}^{c} W_{j,d}^{P}(1+R_{j,t,d}^{P})^{-1}}\left(1-\dfrac{\sum_{k=1}^{m} W_{k,d}^{P}}{\sum_{i=1}^{n} W_{i,d}^{P}}\right) & \text{if s has complete perf in infer wt period} \end{cases}$$

Where:

$birw_{s,t}^{P}$	=	backward-inferred re-scaled portfolio weight for stock s at period t, where t is the date in the Weights Based On column
$W_{s,d}^{P}$	=	original weight of stock s in the portfolio at period d, where d is the date in the Infer From column
$R_{s,d,t}^{P}$	=	portfolio return for stock s in the inferring weight period from d to t, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the portfolio excluding ignored securities and those with incomplete performance in the inferring weight period from d to t
n	=	number of securities in the portfolio except for ignored securities, where $n = c + m$
m	=	number of securities in the portfolio with incomplete performance in the inferring weight period from d to t but excluding ignored securities

Note:

- This formula does not apply to the last single period where weights need to be filled forward using formula [7].

Inferring Weights and Re-Scaling to Exclude Exception Securities (continued)

Re-Scaling and Inferring Benchmark Weights Backward In Time

The methodology for inferring while re-scaling benchmark holdings weights is similar to that of inferring while re-scaling portfolio holdings weights. The formula is below:

[10]

$$\begin{cases} birw^{B}_{s,t} = \text{ignored} & \text{if s has incomplete perf in attribution period or is a non - equity or unrecognized security} \\ \text{otherwise, } birw^{B}_{s,t} = \dfrac{W^{B}_{s,d}}{\sum_{i=1}^{n} W^{B}_{i,d}} & \text{if s has incomplete perf in infer wt period} \\ \text{otherwise, } birw^{B}_{s,t} = \dfrac{W^{B}_{s,d}(1+R^{B}_{s,t,d})^{-1}}{\sum_{j=1}^{c} W^{B}_{j,d}(1+R^{B}_{j,t,d})^{-1}}\left(1-\dfrac{\sum_{k=1}^{m} W^{B}_{k,d}}{\sum_{i=1}^{n} W^{B}_{i,d}}\right) & \text{if s has complete perf in infer wt period} \end{cases}$$

Where:

$birw^{B}_{s,t}$	=	forward-inferred re-scaled benchmark weight for stock s at period t, where t is the date in the Weights Based On column
$W^{B}_{s,d}$	=	original weight of stock s in the benchmark at period d, where d is the date in the Infer From column
$R^{B}_{s,d,t}$	=	benchmark return for stock s in the inferring weight period from d to t, and this period is shown in the Returns Used In Inferring column
c	=	number of securities in the benchmark excluding ignored securities and those with incomplete performance in the inferring weight period from d to t
n	=	number of securities in the benchmark except for ignored securities, where $n = c + m$
m	=	number of securities in the benchmark with incomplete performance in the inferring weight period from d to t but excluding ignored securities

Note:

- This formula does not apply to the first three single periods where weights need to be filled forward using formula [8].

Calculating Multi-Period Average Weights

When an attribution analysis consists of multiple single periods, it is a common industry practice to display holdings weights in average numbers even though these figures are not used in attribution analysis. Holdings weights of all underlying single periods are incorporated in the averaging process, regardless of whether the weights are actual or inferred. In a multi-period analysis, the average weight of a group is the time-weighted average of the group's weight at the beginning of each single period. This method gives more importance to single periods that include more number of days. The formulas for averaging single-period weights are as follow:

[11] $$\overline{w}_g^P = \frac{\sum_{t=1}^{N} d_t w_{g,t}^P}{T}$$

[12] $$\overline{w}_g^B = \frac{\sum_{t=1}^{N} d_t w_{g,t}^B}{T}$$

Where:

$\overline{w}_g^P$	=	average weight of group g in the portfolio
$\overline{w}_g^B$	=	average weight of group g in the benchmark
N	=	number of single periods in the multi-period attribution analysis
d_t	=	number of calendar days in single period t
$w_{g,t}^P$	=	portfolio weight of group g in single period t
T	=	total number of calendar days in the multi-period attribution analysis
$w_{g,t}^B$	=	benchmark weight of group g in single period t

Note:

- g is the vector that defines the group. Refer to the Morningstar® Equity Performance Attribution Methodology for details.
- When a security or a group is not held during a particular single period, its weight is considered to be zero in the period.
- When an exception security is ignored during a particular single period, the weight of that security is considered to be zero in the period.
- In the rare situation where all securities in the portfolio are exceptions and are therefore ignored during a particular single period, the single period is excluded from the attribution analysis and the multi-period average weight calculation.

Fund of Funds

A fund of funds is an investment fund that holds a portfolio of other investment funds rather than individual securities. These funds often report the underlying funds as portfolio holdings instead of the individual securities held in these underlying funds. The methodology in this section applies to tools that provide capabilities that see through the underlying funds to report on aggregate individual securities holdings. This see-through feature provides additional insights on decisions other than asset allocation and manager selection, such as country, sector, and security selections that the fund of funds manager might have made in the selection of the underlying managers. For example, a fund of funds manager that wants to underweight the Technology sector may select or allocate more money to funds that underweight the sector, and the performance impact of this sector allocation decision can be measured in a see-through analysis.

To determine single periods and holdings weights of individual securities, the steps are as follows:

1. Follow the standard methodology described in the Determining Single Periods section of this document, use the holdings dates of the fund of funds, not those of the underlying funds.
2. Since underlying funds may report portfolio holdings on dates different from those of the fund of funds, the portfolio weights of holdings in each individual fund must be inferred to the holdings dates of the fund of funds to achieve uniformity. When inferring the weights, apply the standard inferring and rescaling methodologies described in prior sections.
3. Multiply the weight of each security to the fund's proportion in the fund of funds in order to reach the security's weight in the overall portfolio.
4. For display purpose, when the same security is in multiple underlying funds, the weights are aggregated to form a single weight for the security.

Aggregate

An aggregate is a portfolio of investments or indexes created by assigning a weight to each investment. The underlying investments may be securities, funds, or fund of funds. The methodology in this section applies to tools that provide capabilities that see through the underlying funds or fund of funds to report on aggregate individual securities holdings.

Holdings weights of an aggregate is influenced by two factors. An aggregate has portfolio holdings that are updated from time to time. In addition to these updates, holdings weights of investments in an aggregate may be reset periodically based on a rebalancing methodology. Since portfolio holdings are updated by the user, the timing of these updates ought to reflect relevant decision making in order to achieve a meaningful attribution analysis.

To determine single periods and holdings weights of individual securities, the steps are as follows:

1. Follow the standard methodology described in the Determining Single Periods section of this document, and use the holdings dates of the aggregate and not those of the underlying funds.
2. Since underlying funds may report portfolio holdings on dates different from those of the aggregate, the portfolio weights of holdings in each individual fund must be inferred to the holdings dates of the aggregate to achieve uniformity. When inferring the weights, apply the standard inferring and rescaling methodologies described in prior sections.
3. Multiply the weight of each security to the fund's proportion in the aggregate in order to reach the security's weight in the overall portfolio.
4. For display purpose, when the same security is in multiple underlying funds, the weights are aggregated to form a single weight for the security.